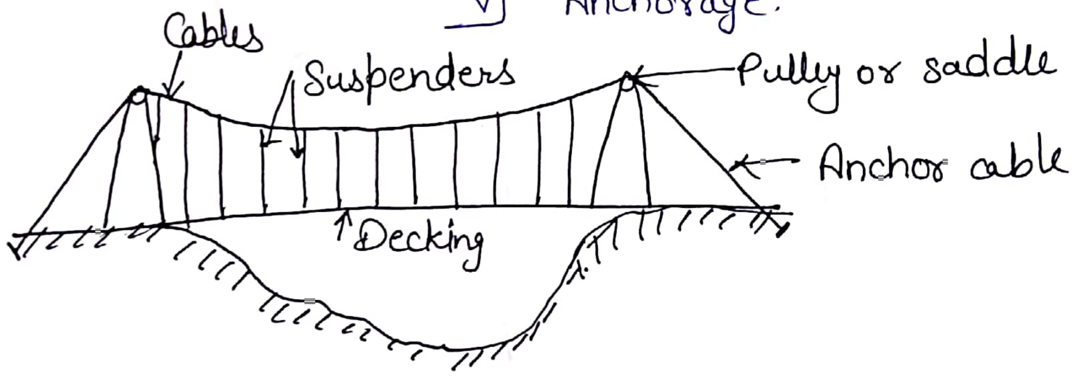
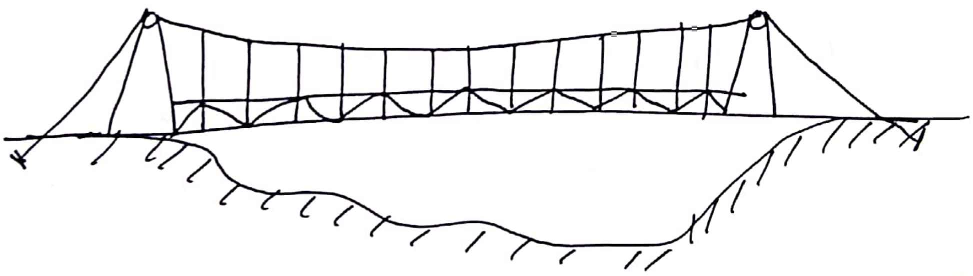
UNIT — 4

CABLE & SUSPENSION BRDGES

Introduction:

- Suspension bridges are used for highways, where the span of bridge is more than 200m.
- Essentially, a suspension bridge consists of following element
  - I) Cable
  - II) Suspenders
  - III) Decking including stiffening girder.
  - IV) Supporting tower
  - V) Anchorage.

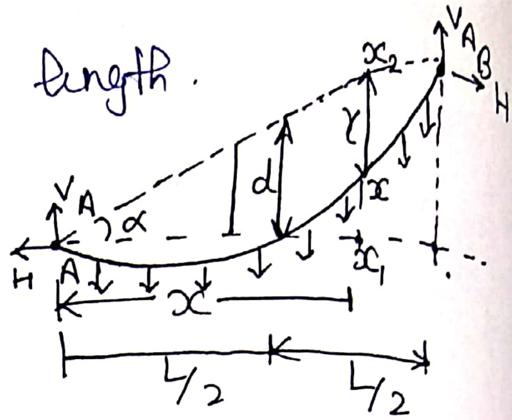




- The traffic load of the decking is tranferred to main cable through suspenders. Since the cable is the main load bearing member, the curvature of the cable of an unstiffened bridge changes as the load moves on the decking.

- Uniformly Loaded cables :-

a) Expression for Horizontal Reaction :-



Load intensity p per unit length.

General cable theorem

$$Hy = \frac{x}{L}\Sigma M_B - \Sigma M_x$$

$y = XX_2$ = vertical ordinate b/w the line AC and chord at the point X.

$$\Sigma M_B = pL \times \frac{L}{2} = \frac{pL^2}{2}$$

$$\Sigma M_x = px \times \frac{x}{2} = \frac{px^2}{2}$$

$$Hy = \frac{x}{L} \times \frac{pL^2}{2} - \frac{px^2}{2} = \frac{pLx}{2} - \frac{px^2}{2}$$

At mid span $x = L/2$ and $y = d = d/p$ of cable

$$Hd = \frac{pL}{2} \times \frac{L}{2} - \frac{p}{2}\left(\frac{L}{2}\right)^2 = \frac{pL^2}{8}$$

$$\boxed{H = \frac{pL^2}{8d}}$$

b) Expression for cable Tension at the ends :-

The cable tension T at any end is the resultant of vertical & Horizontal reaction at the end.

$$T_A = \sqrt{V_A^2 + H^2}$$

$$T_B = \sqrt{V_B^2 + H^2}$$

$$H = \frac{pL^2}{8d} \qquad V_A = V_B = \frac{pL}{2}$$

$$T_A = T_B = T = \sqrt{\left(\frac{pL}{2}\right)^2 + \left(\frac{pL^2}{8d}\right)^2}$$

$$= \frac{pL}{2}\sqrt{1 + \frac{L^2}{16d^2}}$$

$$T = H\sqrt{1 + \frac{L^2}{16d^2}}$$

Inclination β

$$\tan\beta = \frac{H}{V} = \frac{pL^2}{8d} \cdot \frac{2}{pL} = \frac{L}{4d}$$

→ Horizontal component of cable tension at any point will be equal to H.

c) <u>Shape of cable</u> ÷

- Let us determine the shape of cable under the uniformly distribute load.

$$Hy = \frac{pLx}{2} - \frac{px^2}{2}$$

$$\left[\frac{pL^2}{8d}\right] y = \frac{4dx}{L^2}(L-x) \qquad \text{Parabolic eq}^n.$$

d) <u>Length of cable</u> : Both ends at the same level

- Equation of parabla

$$y = kx^2$$

$$x = \frac{L}{2} \quad \& \quad y = d$$

$$k = \frac{y}{x^2} = \frac{d}{(L/2)^2} = \frac{4d}{L^2}$$

$$y = \frac{4d}{L^2}x^2$$

$$\boxed{\frac{dy}{dx} = \frac{8d}{L^2}x}$$

Consider an element of length ds of the curve, having co-ordinates x & y. The total length s of the curve is given by

$$s = \int_0^L ds = 2\int_0^{L/2} \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} dx$$

$$= 2\int_0^{L/2} \left(1 + \frac{64d^2}{L^4}x^2\right)^{1/2} dx$$

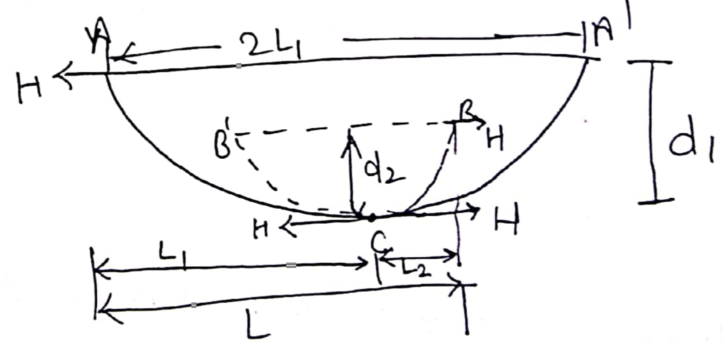
Expanding $\left(1 + \frac{64d^2}{L^4}x^2\right)$ by Binomial theorem and neglecting higher power of $\frac{d^2}{L^4}x^2$ we get

$$s = 2\int_0^{L/2} \left(1 + \frac{1}{2}\frac{64d^2}{L^4}x^2 + \cdots\right) dx$$

$$= 2\left[x + \frac{32d^2}{3L^4}x^3\right]_0^{L/2} = 2\left[\frac{L}{2} + \frac{4}{3}\frac{d^2L^3}{L^4}\right]$$

$$\boxed{s = L + \frac{8}{3}\frac{d^2}{L}}$$

e) Length of cable : Ends at different levels



$$L = L_1 + L_2$$

<u>AA'</u>

$$H = \frac{pL^2}{8d} \qquad L = 2L_1 \quad \& \quad d = d_1$$

$$H = \frac{p}{8}\left[\frac{2L_1^2}{d_1}\right] = \frac{pL_1^2}{2d_1}$$

BB'

$$H = \frac{pL^2}{8d} \qquad L = 2L_2 \qquad d = d_2$$

$$H = \frac{p}{8}\left[\frac{2L_2}{d_2}\right]^2 = \frac{pL_2^2}{2d_2}$$

Since H is the same at C for both the portion of cable we get

$$\frac{pL_1^2}{2d_1} = \frac{pL_2^2}{2d_2}$$

$$\boxed{\frac{L_1}{L_2} = \sqrt{\frac{d_1}{d_2}}}$$

- **Temperature stresses in Suspension cable :-**

S = length of cable

∂S = change in length due to change in temperature

∂d = corresponding change in dip

$$S = L + \frac{8d^2}{3L}$$

$$\partial S = \frac{16d\,\partial d}{3L}$$

$$\boxed{\partial d = \frac{3L\,\partial S}{16d}} \rightarrow (1)$$

α = coff. of thermal expn

t = change temperature

$$\boxed{\text{But} \quad \partial S = S\alpha t}$$

$$\partial S = \alpha t\left(L + \frac{8d^2}{3L}\right)$$

$$\partial S = L\alpha t + \frac{8d^2\alpha t}{3L}$$

Neglecting $\frac{8d^2\alpha t}{3L}$ in comparison to $L\alpha t$

we have $\partial S = L\alpha t \rightarrow (2)$

Substituting in (1)

$$\partial d = \frac{3L}{16d}(L\alpha t) = \frac{3L^2}{16d}\alpha t$$

we know $H = \frac{pL^2}{8d}$

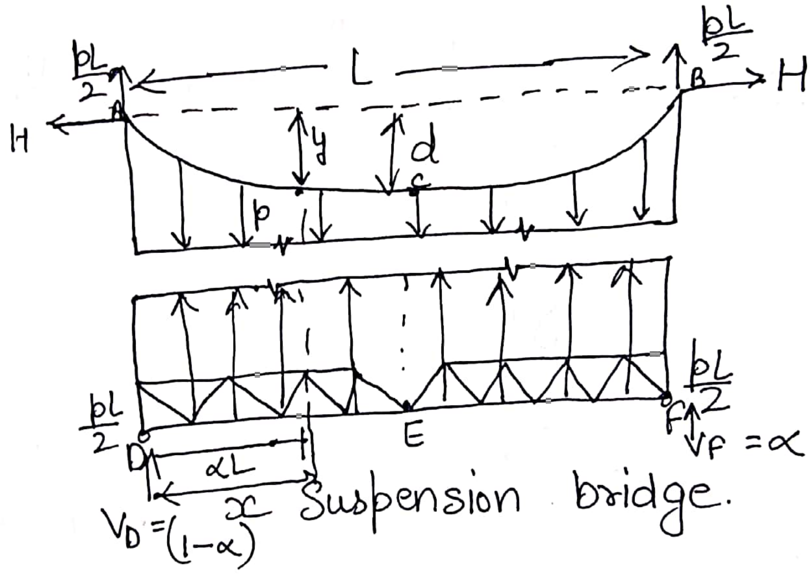
$$H \propto \frac{1}{d}$$

$$\frac{\partial H}{H} = \frac{-\partial d}{d}$$

If f is stress in cable

$$\boxed{\frac{\partial f}{f} = \frac{\partial H}{H} = \frac{-\partial d}{d} = -\frac{3L^2\alpha t}{16d^2}}$$

- <u>Three hinged Stiffening Girder</u> :-



Suspension bridge.

a) <u>Equilibrium of the cable</u> :-

$$H = \frac{pL^2}{8d}$$

$$Hy = \frac{4dx}{L^2}[L-x]$$

$$y = kx(L-x)$$

$$x = \frac{L}{2}, \ y = d$$

$$k = \frac{4d}{L^2}$$

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b] Equilibrium of girder :

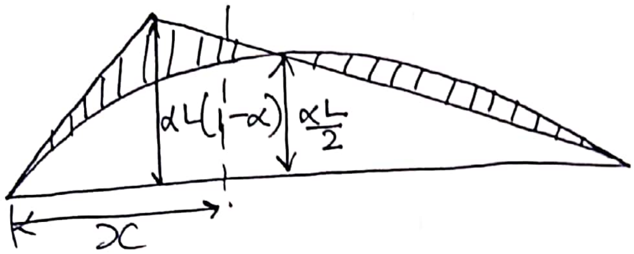
i] External $\overset{\text{unit}}{\text{load}}$ acting at a distance of αL from left hand support.

ii] Reaction $V_F = (1-\alpha)$ & $V_D = \alpha$

iii] UDL, pull p exterted by suspender

iv] downward reaction $\frac{pL}{2}$ at D & F due to p at hinger.

c] Bending moment Diagram :



• consider Pt X at distance from x where BM is expressed as :-

$$M_x = \left[-V_D x + 1(x-\alpha L)\right] + \left[\frac{pL}{2}x - \frac{px^2}{2}\right]$$

$$\boxed{M_x = \mu_x + Hy}$$

• The μ_x diagram for simply supported beam is a triangle having ordinate of

$$-\frac{Wab}{L} = -\frac{1 \times \alpha L[1-\alpha]L}{L} = -\alpha L[1-\alpha]$$

$$\boxed{H = \frac{\alpha L}{2d}} \qquad \boxed{d = \frac{\alpha L}{2}}$$

Hy diagram is parabolic having max ordinate

d) Influence Line for H :

- In order to find the value of H for the unit load position at a distance αL, we apply at central hinge E of the girder where bending moment is zero.

$$M_E = 0 = \mu_E = Hy$$

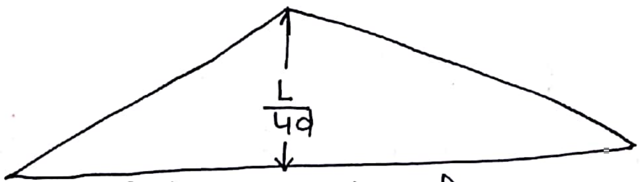
$$H = -\frac{\mu_E}{y}$$

$$\mu_E = -\frac{\alpha L}{2} \qquad \text{and } y = d \text{ at } E$$

$$\boxed{H = \frac{\alpha L}{2d}}$$

when $\alpha L = 0$ $\qquad H = 0$

when $\alpha L = \frac{H}{2}$ $\qquad H = \frac{L}{4d}$



Influence line for H

e) Influence Line for p :

$$H = \frac{pL^2}{8d} \qquad p = \frac{8d}{L^2} H$$

$$H = \frac{\alpha L}{2d}$$

$$p = \frac{8d}{L^2} \times \frac{\alpha L}{2d} = \frac{4\alpha}{L}$$

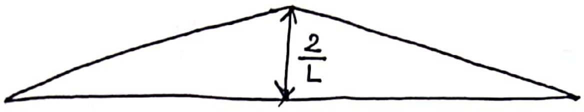
- Thus the load carried by suspenders vary with the load position in the case of three hinged stiffening girder.

Range → $\alpha L = 0$ to $\alpha L = \frac{L}{2}$

- When load is at D, $\alpha = 0$ $\therefore p = 0$
- When load is at central hinge E, $\alpha L = \frac{L}{2}$ or $\alpha = \frac{L}{2}$

$$p = \frac{4}{L} \times \frac{1}{2} = \frac{2}{L}$$

- By symmetry when load is at F, $p = 0$.



Influence line for p

f) Maximum Bending moment diagram due to single point load W.

i) Maximum negative BMD :-

$$M_{x\,(max\,-ve)} = W\left[-\frac{x(L-x)}{L} + \frac{x}{L}(L-x)\frac{2x}{L}\right]$$

$$= \frac{Wx}{L^2}\left[-L^2 + Lx + 2Lx - 2x^2\right]$$

$$= -\frac{Wx}{L^2}\left[L-x\right]\left[L-2x\right]$$

- the value of x is third degree polynomial
- To find the position of section where absolute maximum negative BM occurs, differentiate above eqn w.r.t x & equate it to zero

$$[L-x](L-2x) - x(L-2x) - 2x(L-x) = 0$$

$$6x^2 - 6Lx - L^2 = 0$$

$$x = 0.211L \quad \text{or} = 0.789L$$

$$\boxed{-[M_{x\,max}] = -0.096WL}$$

ii b) Maximum positive BMD :

$$M_x(\max +ve) = W\left[\frac{x}{L}(L-x) - \frac{x(L-x)}{L}\times\frac{L}{L-x}\cdot\frac{L}{2}\right]$$

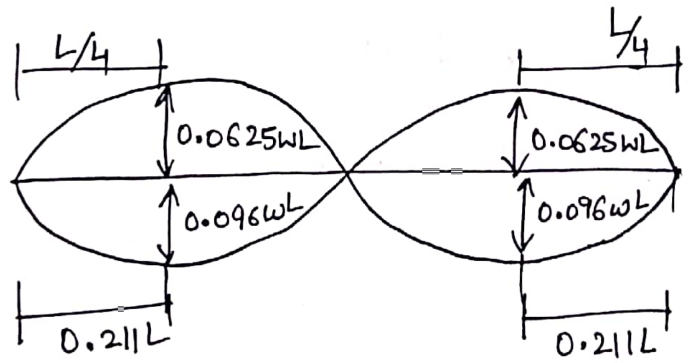
$$= \frac{Wx}{2L}(L-2x)$$

which is eqn of parabola

$$\frac{dM_x}{dx} = 0 = (L-2x) - 2x$$

$$x = \frac{L}{4}$$

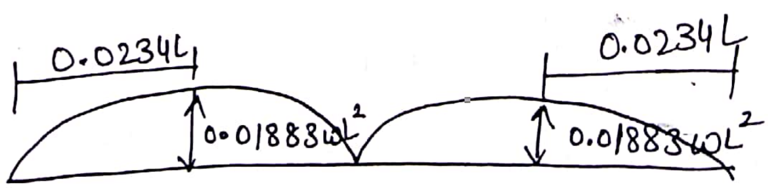
$$\boxed{+(M_{x\,max}) = \frac{WL}{16}} \quad \text{or} = 0.0625WL$$



g) Maximum BMD due to UDL :

$$(\pm) M_{max} = \pm\, 0.01883\, wL^2$$

$$x = 0.0234\, L$$



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h) Influence line ~~for~~ for Shear Force +

$$F_x = +\alpha + p(L-x) - \frac{pL}{2}$$

$$F_x = \alpha + p\left[\frac{L}{2} - x\right]$$

$$F_x = f_x + p\left[\frac{L}{2} - x\right]$$

$\alpha = f_x$ = SF at X by considering the girder to be simply supported

$$y = \frac{4d}{L^2} x(L-x)$$

$$\tan\theta = \frac{dy}{dx} = \frac{8d}{L^2}\left(\frac{L}{2} - x\right)$$

θ = inclination of tangen

But $H = \frac{pL^2}{8d}$ or $\frac{8d}{L^2} = \frac{p}{H}$

$$\tan\theta = \frac{p}{H}(L-x)$$

$$p\left(\frac{L}{2} - x\right) = H\tan\theta$$

$$\boxed{F_x = f_x + H\tan\theta}$$

$$H = \frac{\alpha L}{2d} = \frac{L}{4d}$$

$$H\tan\theta = \frac{L}{4d} \cdot \frac{8d}{L^2}\left(\frac{L}{2} - x\right)$$

$$= \frac{2}{L}\left(\frac{L-2x}{2}\right) = \left(1 - \frac{2x}{L}\right)$$

g) SFD

- $F_x = -(1-\alpha) + \frac{pL}{2} - px + 1$

$$p = \frac{4\alpha}{L}$$

- $F_x = -(1-\alpha) + \frac{4\alpha}{L}\left(\frac{L}{2} - x\right) + 1$

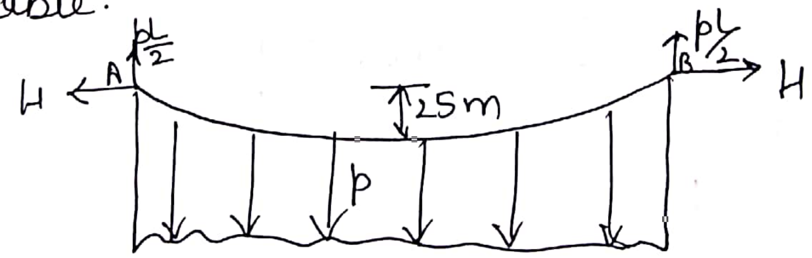
$x = 0$, $F = -(1-\alpha) + 2\alpha = (-1+3\alpha)$

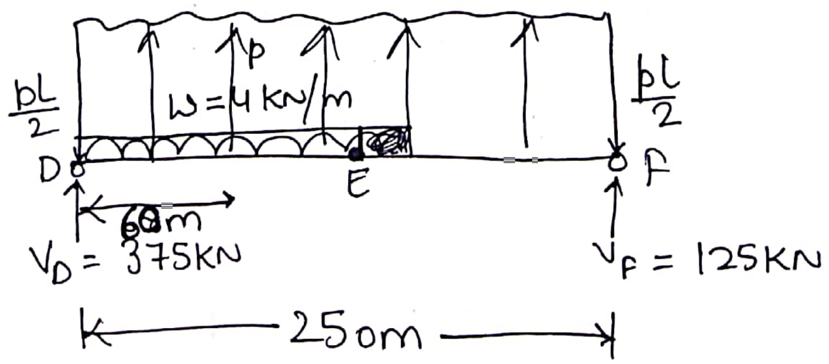
$x = \alpha L$, $F = -1 + 3\alpha - 4\alpha^2 = -3a - 4a^2$

$x = L$, $F = -(1-\alpha) - 2\alpha + 1 = -\alpha$

Q-1 A suspension bridge of 250m span has two three hinged stiffning girder supported by two cables having a central dip of 25m. The width of roadway is 8m. The roadway carries a dead Load of 1 KN/m² extending over whole span and a live load of 1KN/m² extending over left hand half of bridge. Find BM & SF at point 60m & 200m from left hinge. Also calculate the maximum tension in the cable.

Sol





The free BD for cable & stiffening girder. Since there will be no BM & SF anywhere in the girder due to UDL, only Live load has shown.

Live load per meter run by girder

$$w = [\cancel{15}4 \times 1 \times 1] = 4 \text{ kNm}$$

$$V_F = \frac{1}{250}(4 \times 125 \times 62.5) = 125 \text{ KN}$$

$$M_E = 3 = \mu_E + Hy$$

$$(-125 \times 125) + 25H = 0$$

$$H = \frac{125 \times 125}{25} = 625 \text{ KN}$$

- Equation of cable

$$y = \frac{4d}{L^2} x(L-x) = \frac{4 \times 25}{250 \times 250} x(250-x)$$

$$x = \frac{x}{625}(250-x)$$

$$\frac{dy}{dx} = \tan\theta = \frac{250-2x}{625}$$

At $x = 60m$, $y = \frac{60}{625}(250-60) = 18.25 m$

$$\tan\theta = \frac{250-120}{625} = 0.208$$

At $x = 200m$, $y = \frac{200}{625}(500-200) = 16$

$$\tan\theta = \frac{250-400}{625} = 0.24$$

$$M_{60} = \mu + Hy = \left(\left(-375 \times 60\right) + \left(\frac{4 \times 60 \times 60}{2}\right)\right] + (625 \times 18.25)$$

$$= -3894 \text{ kNm}$$

$$M_{200} = (-125 \times 50) + (625 \times 16) = 3750 \text{ kNm}$$

$$F_{60} = f_{c0} + H\tan\theta$$

$$= (-375 + (60\times4)) + (625\times0.208) = -5\,kN$$

$$F_{200} = 125 + (625\times(-0.24)) = -25\,kN$$

- Again the equivalent UDL transferred to the cable due to live load

$$\frac{8d}{L^2}H = \frac{8\times25}{240\times250}\times625 = 2\,kN/m$$

- Equivalent UDL transferred due to dead load per meter run.

$$(2\times1\times1) = 2\,kN/m$$

$$\text{Total} = p = 2+2 = 4\,kN/m$$

- Maximum tension in the cable:

$$T = \frac{pL}{2}\sqrt{1+\frac{L^2}{16d^2}}$$

$$= \frac{4\times250}{2}\sqrt{1+\frac{250\times250}{16\times25\times25}} = \underline{\underline{1365\,kN}}$$

Q.2 A suspension cable, stiffened with a three hinged girder, has 100m span, 10m dip. The girder carries a load of 0.4 kN/m.
A live load of 10 kN rolls from left to right. Determine max BM anywhere in the girder.

Sol $x = 0.211\,L = 0.211 \times 100 = 21.1\,m$

$(-)\,M_{max} = -0.096\,WL = 0.096 \times 10 \times 100 = 96\,KN\text{-}m$

Absolute max positive BM

$x = 0.25L = 25m$

$(+M_{max}) = 0.0625WL = 0.0625 \times 10 \times 100$
$= 62.5\,kN\text{-}m$

• Two Hinged Stiffening girder :

a) Influence Line for H :

$$H = \frac{pL^2}{8d}$$

$$p = \frac{W}{L} = \frac{1}{L}$$

$$H = \frac{L^2}{L.8.d} = \frac{L}{8d}$$

b) BMD :

$$M_x = \mu_x + Hy$$

$$\mu_x = \frac{\alpha L(L-\alpha L)}{L} = \alpha L(L-\alpha)$$

$$Hy = \frac{L}{8d} \times d = \frac{L}{8}$$

$$y = \frac{L}{8d} \cdot \frac{4d_x}{L^2}(L-x) = \frac{x}{2L}(L-x)$$

c) **Max BMD due to single load W :-**

$$\pm M_{max} = \frac{WL}{8}$$

d) **Max BMD due to UDL :-**

$$\pm M_{max} = \frac{wL^2}{32}$$

e)

- **Temperature Stress in two Hinged girder :-**

$$\delta d = \frac{3}{16}\frac{L^2}{d}\alpha t$$

Due to change in δp, the change in max deflection at the centre of girder is given by.

$$\delta(\Delta) = \cancel{\frac{5}{384}\delta p} \; \frac{5}{384}\frac{\delta P L^4}{EI}$$

$$\delta\Delta = \delta d$$

$$\frac{5}{384}\frac{\delta P L^4}{EI} = \frac{3}{16}\frac{L^2}{d}\alpha t$$

$$\delta P = \frac{72}{5}\frac{EI\alpha t}{L^2 d}$$

Increase in BM at girder $= \frac{\delta P \cdot L^2}{8}$

Increase in stress in girder $= \frac{\delta P L^2}{8} \times \frac{D}{2I}$

$$= \frac{72}{5}\frac{EI\alpha t}{L^2 d} \times \frac{L^2 D}{16 I}$$

$$= \frac{9}{10}\frac{D}{d} E\alpha t$$

Q.1 A suspension bridge with two hinged stiffening girder has a span of 100m & the cables have a central dip of 10m. The stiffening girder are 4m deep & have moment of inertia equal to $1.64 \times 10^{10}\ mm^4$. If temp falls through 22 Kelvin calculate i) flange stress
ii) Increase in tension in cable
Take $E = 2 \times 10^6\ N/mm^2$ & $\alpha = 11 \times 10^{-6}$ per 1K.

Sol :- • Stress in flange due to fall in temp.

$$\frac{9}{10}\frac{D}{d}E\alpha t = \frac{9}{10} \times \frac{4}{10} \times 2 \times 10^6 (11 \times 10^{-6}) \times 22$$
$$= 17.42\ N/mm^2$$
$$= 17.42 \times 10^{-3}\ kN/mm^2$$

• The increase in load δp on the girder is given by

$$\delta p = \frac{72}{5} EI \frac{\alpha t}{L^2 d}$$

$$EI = (2 \times 10^6)(1.64 \times 10^{10}) = 3.28 \times 10^{16}\ N\text{-}mm^2$$
$$=$$

$$\delta p = \frac{72}{5}(3.28 \times 10^6)\frac{11 \times 10^{-6} \times 22}{100 \times 100 \times 10} = 0.1143\ kN\text{-}m$$

$$\delta H = \frac{\delta p L^2}{8d} = \frac{0.1143 \times 100^2}{8 \times 10} = 14.29\ kN$$

• Change in cable tension

$$\delta T = \delta H \sqrt{1 + \left(\frac{4d}{L}\right)^2}$$
$$= 14.29 \sqrt{1 + \left(\frac{4 \times 10}{100}\right)^2} = 15.39\ kN$$

Q.2 A suspension cable has a span of 160m & a central dip of 16m, and its suspended from the same level at both towers. The bridge is stiffened by a stiffening girder hinged at the end supports. The girder carries a single concentrated load of 8KN at a point 40m from left end. Assuming equal tensions in the suspension hangers calculate i) Horizontal tension & ii) Max positive & -ve BM.

If the 8KN load rolls from left to right, what will be the value of absolute max BM & SF & where do they occur?

i) Load per meter run in hangers $= p = \frac{W}{L}$

$$= \frac{8}{160} = 0.05 \text{ KN/m}$$

Horizontal tension $= \frac{pL^2}{8d} = \frac{0.05 \times 160 \times 160}{8 \times 16} = \underline{\underline{10KN}}$

ii) Simply supported reactions are :-

$$V_F = \frac{8 \times 40}{160} = 2KN$$

$$V_D = 8 - 2 = 6KN$$

iii) Max -ve BM

$$(-)(M_{max} = \left(-V_D \times 40\right) + \left(\frac{pL}{2} \times 40\right) - \frac{p}{2}(40)^2$$

$$= -240 + \frac{0.05 \times 160 \times 40}{2} - \frac{0.05}{2} \times 1600$$

$$= -120 \text{ KNm}$$

iv] <u>Max +ve BM</u>

$$M = -V_F x + \frac{pL}{2} x - \frac{p}{2} x^2$$

$$= -2x + \frac{0.05 \times 160}{2} x - \frac{0.05}{2} x$$

$$= 2x - \frac{0.05}{2} x^2$$

For maxma $\frac{dM}{dx} = 0 = 2 - 0.05x$

$$x = \frac{2}{0.05} = 40m$$

$$(+M_{ax}) = 2x - \frac{0.05}{2}x^2 = (2 \times 40) + \left(\frac{0.05}{2}(40)^2\right)$$

$$= 40 kNm$$

v] Absolute maximum ($\pm$) BM occurs at mid span

$$(\pm M_{max}) = \frac{WL}{8} = \frac{8 \times 160}{8} = \underline{160 kNm}$$